

ON THE FINAL STATE OF A SYSTEM OF MOLECULES IN MOTION SUBJECT TO FORCES OF ANY KIND

LET perfectly elastic molecules of different kinds be in motion within a vessel with perfectly elastic sides, and let each kind of molecules be acted on by forces which have a potential, the form of which may be different for different kinds of molecules.

Let x, y, z , be the the coordinates of a molecule, M , and ξ, η, ζ the components of its velocity, and let it be required to determine the number of molecules of a given kind which, on an average, have their coordinates between x and $x + dx$, y and $y + dy$, z and $z + dz$, and also their component velocities between ξ and $\xi + d\xi$, η and $\eta + d\eta$ and ζ and $\zeta + d\zeta$. This number must depend on the coordinates and the components of velocities and on the limits of these quantities. We may therefore write it

$$dN = f(x, y, z, \xi, \eta, \zeta) dx dy dz d\xi d\eta d\zeta \quad (1)$$

We shall begin by investigating the manner in which this quantity depends on the components of velocity, before we proceed to determine in what way it depends on the coordinates.

If we distinguish by suffixes the quantities corresponding to different kinds of molecules, the whole number of molecules of the first and second kind within a given space which have velocities within given limits may be written

$$f_1(\xi_1, \eta_1, \zeta_1) d\xi_1 d\eta_1 d\zeta_1 = n_1 \quad (2)$$

and

$$f_2(\xi_2, \eta_2, \zeta_2) d\xi_2 d\eta_2 d\zeta_2 = n_2 \quad (3)$$

The number of pairs which can be formed by taking one molecule of each kind is $n_1 n_2$.

Let a pair of molecules encounter each other, and after the encounter let their component velocities be $\xi_1', \eta_1', \zeta_1'$ and $\xi_2', \eta_2', \zeta_2'$. The nature of the encounter is completely defined when we know $\xi_2 - \xi_1, \eta_2 - \eta_1, \zeta_2 - \zeta_1$ the velocity of the second molecule relative to the first before the encounter, and $x_2 - x_1, y_2 - y_1, z_2 - z_1$ the position of the centre of the second molecule relative to the first at the instant of the encounter. When these quantities are given, $\xi_2' - \xi_1', \eta_2' - \eta_1', \zeta_2' - \zeta_1'$, the components of the relative velocity after the encounter, are determinable.

Hence, putting a, b, γ for these relative velocities, and a, b, c for the relative positions, we find for the number of molecules of the first kind having velocities between the limits ξ_1 and $\xi_1 + d\xi_1$, &c., which encounter molecules of the second kind having velocities between the limits ξ_2 and $\xi_2 + d\xi_2$, &c., in such a way that the relative velocities lie between a and $a + da$, &c., and the relative positions between a and $a + da$, &c.

$f_1(\xi_1, \eta_1, \zeta_1) d\xi_1 d\eta_1 d\zeta_1 \cdot f_2(\xi_2, \eta_2, \zeta_2) d\xi_2 d\eta_2 d\zeta_2 \cdot \phi(a, b, c) da db dc d\alpha d\beta d\gamma \quad (4)$ and after the encounter the velocity of M_1 will be between the limits $\xi_1' + d\xi_1'$ and $\xi_1' + d\xi_1'$, &c., and that of M_2 between the limits $\xi_2' + d\xi_2'$ and $\xi_2' + d\xi_2'$, &c.

The differences of the limits of velocity are equal for both kinds of molecules, and both before and after the encounter.

When the state of motion of the system is in its permanent condition, as many pairs of molecules change their velocities from V_1, V_2 to V_1', V_2' as from V_1', V_2' to V_1, V_2 , and the circumstances of the encounter in the one case are precisely similar to those in the second. Hence, omitting for the sake of brevity the quantities $d\xi$, &c., and ϕ , which are of the same value in the two cases, we find—

$$f_1(\xi_1, \eta_1, \zeta_1) f_2(\xi_2, \eta_2, \zeta_2) = f_1(\xi_1', \eta_1', \zeta_1') f_2(\xi_2', \eta_2', \zeta_2') \quad (5)$$

writing—

$$\log f(\xi, \eta, \zeta) = F(MV^2, l, m, n) \quad (6)$$

where l, m, n are the direction cosines of the velocity, V , of the molecule M .

Taking the logarithm of both sides of equation (5)—

$$F(M_1 V_1^2, l_1, m_1, n_1) + F(M_2 V_2^2, l_2, m_2, n_2) = F(M_1 V_1'^2, l_1', m_1', n_1') + F(M_2 V_2'^2, l_2', m_2', n_2') \quad (7)$$

The only necessary relation between the variables before and after the encounter is—

$$M_1 V_1^2 + M_2 V_2^2 = M_1 V_1'^2 + M_2 V_2'^2 \quad (8)$$

If the righthand side of the equations (7) and (8) are constant, the lefthand sides will also be constant; and since l_1, m_1, n_1 are independent of l_2, m_2, n_2 we must have—

$$F_1 = A M_1 V_1^2 \text{ and } F_2 = A M_2 V_2^2 \quad (9)$$

where A is a quantity independent of the components of velocity, or—

$$f_1(\xi_1, \eta_1, \zeta_1) = C_1 e^{A M_1 V_1^2} \quad (10)$$

$$f_2(\xi_2, \eta_2, \zeta_2) = C_2 e^{A M_2 V_2^2} \quad (11)$$

This result as to the distribution of the velocities of the molecules at a given place is independent of the action of finite forces on the molecules during their encounter, for such forces do not affect the velocities during the infinitely short time of the encounter.

We may therefore write equation (1)

$$dN = C e^{A M(\xi^2 + \eta^2 + \zeta^2)} d\xi d\eta d\zeta dx dy dz \quad (12)$$

where C is a function of x, y, z which may be different for different kinds of molecules, while A is the same for every kind of molecule, though it may, for aught we know as yet, vary from one place to another.

Let us now suppose that the kind of molecules under consideration are acted on by a force whose potential is ψ . The variations of x, y, z arising from the motion of the molecules during a time δt are

$$\delta x = \xi \delta t, \delta y = \eta \delta t, \delta z = \zeta \delta t \quad (13)$$

and those of ξ, η, ζ in the same time due to the action of the force, are

$$\delta \xi = -\frac{d\psi}{dx} \delta t, \delta \eta = -\frac{d\psi}{dy} \delta t, \delta \zeta = -\frac{d\psi}{dz} \delta t \quad (14)$$

If we make

$$c = \log C \quad (15)$$

$$\log \frac{dN}{d\xi d\eta d\zeta dx dy dz} = c + A M(\xi^2 + \eta^2 + \zeta^2) \quad (16)$$

The variation of this quantity due to the variations $\delta x, \delta y, \delta z, \delta \xi, \delta \eta, \delta \zeta$ is

$$\left\{ \begin{aligned} & \left(\xi \frac{dc}{dx} + \eta \frac{dc}{dy} + \zeta \frac{dc}{dz} \right) \delta t \\ & - 2 A M \left(\xi \frac{d\psi}{dx} + \eta \frac{d\psi}{dy} + \zeta \frac{d\psi}{dz} \right) \delta t \\ & + M(\xi^2 + \eta^2 + \zeta^2) \left(\xi \frac{dA}{dx} + \eta \frac{dA}{dy} + \zeta \frac{dA}{dz} \right) \delta t \end{aligned} \right\} \quad (17)$$

Since the number of the molecules does not vary during their motion, this quantity is zero, whatever the values of ξ, η, ζ . Hence we have in virtue of the last term—

$$\frac{dA}{dx} = 0, \frac{dA}{dy} = 0, \frac{dA}{dz} = 0 \quad (18)$$

or A is constant throughout the whole region traversed by the molecules.

Next, comparing the first and second terms, we find

$$c = 2 A M(\psi + B) \quad (19)$$

We thus obtain as the complete form of dN

$$dN_1 = e^{A M_1(\xi_1^2 + \eta_1^2 + \zeta_1^2 + 2\psi_1 + B_1)} dx dy dz d\xi d\eta d\zeta \quad (20)$$

when A is an absolute constant, the same for every kind of molecule in the vessel, but B_1 belongs to the first kind only. To determine these constants, we must integrate this quantity with respect to the six variables, and equate the result to the number of molecules of the first kind. We must then, by integrating $dN_1 \frac{1}{2} M_1(\xi_1^2 + \eta_1^2 + \zeta_1^2 + 2\psi_1)$ determine the whole energy of the system, and equate it to the original energy. We shall thus obtain a sufficient number of equations to determine the constant A , common to all the molecules, and B_1, B_2 , &c. those belonging to each kind.

The quantity A is essentially negative. Its value determines that of the mean kinetic energy of all the molecules in a given place, which is $-\frac{3}{2} \frac{1}{A}$, and therefore, according to the kinetic theory, it also determines the temperature of the medium at that place. Hence, since A_1 , in the permanent state of the system, is the same for every part of the system, it follows that the temperature is everywhere the same, whatever forces act upon the molecules.

The number of molecules of the first kind in the element $dx dy dz$.

$$\left(-\frac{\pi}{A} \right)^{\frac{3}{2}} e^{A M_1(2\psi_1 + B)} dx dy dz \quad (21)$$

The effect of the force whose potential is ψ_1 is therefore to cause the molecules of the first kind to accumulate in greater numbers in those parts of the vessel towards which the force acts, and

the distribution of each different kind of molecules in the vessel is determined by the forces which act on them in the same way as if no other molecules were present. This agrees with Dalton's doctrine of the distribution of mixed gases.

J. CLERK-MAXWELL

ORIGINAL RESEARCH AS A MEANS OF EDUCATION*

THE subject of the value of original scientific investigation may be considered from many points of view. Of these, that of the national importance of original research is the one which naturally first engages attention; and it does not take long to convince us that almost every great material advance in modern civilisation is due, not to the occurrence of haphazard or fortuitous circumstances, but to the long-continued, and disinterested efforts of some man of science. Nor do I need to quote many examples to show us the immediate dependence of the national well-being and progress upon scientific discoveries thus patiently and quietly made. If it had not been for Black's researches on the latent heat of steam, James Watt's great discovery, which has revolutionised the world, would not have been made. Practical applications cannot be made until the scientific facts or principles upon which those applications rest have been discovered. In our own science I might instance hundreds of cases in which discoveries made in the pure spirit of scientific inquiry have (generally in the hands of others than the original investigators) led to results of the first importance to civilisation. Chloroform was first prepared by Liebig in 1834; but it was Simpson who long afterwards applied it to the relief of suffering humanity. Faraday in 1825 discovered benzole, and from it Zinin prepared a substance called aniline, which for many years remained a chemical curiosity only interesting to the scientific man. In due course, however, a practical sphere of usefulness was to be opened out for this little known substance. Perkin discovered that this rare body was capable of yielding splendid colours. Commercial skill then at once seized upon aniline, and, instead of its being made by the ounce, it is now manufactured by thousands of tons, and the bright and beautiful colours which it yields are known all the world over, and are alike pleasing to the eye of the connoisseur of fashion and of the dusky denizen of the forest primæval. Thus, too, the purely scientific researches of our distinguished fellow-citizen Dr. Schunck, respecting the dyeing principle contained in the well-known madder root, laid the foundations for the subsequent discovery, by Graebe and Lieberman, of the artificial production of this naturally occurring principle, termed alizarine, the manufacture of which is now assuming such gigantic proportions. Again, the discovery of chlorine by Scheele, in 1774, lies at the foundation of the whole of our Lancashire trade, for without bleaching powder the cotton and paper manufactures could not exist on their present extended scale. I might almost indefinitely extend this list of discoveries, which, when first made, were apparently far removed from any useful application, but which all at once become the starting-point of a new branch of industry, and a source of benefit or gratification to mankind.

This subject of the national importance of original research is one which is gradually but surely forcing itself on public attention. A few years ago national elementary education was looked upon as a chimera; now it has become the question of the day. As soon as English people see as clearly as we do the imperious necessity for encouraging, stimulating, and upholding original research as containing the seeds of our future position as a nation, they will not be behindhand in securing the free growth of those seeds. It is therefore the bounden duty of all those whose employment or disposition has led them to feel the truth of this great principle, to leave no stone unturned to make widely known and keenly felt the importance of the national encouragement of original investigation.

It might have been a useful task for me to contrast what is done in other countries for the encouragement of free inquiry and research, and what is done, or rather left undone, in England. We should have seen that on the Continent of Europe, to a great extent, and in the United States, in some measure, those who have to wield the sceptre of government are not only aware of the national importance of original research, but, what is more, that they act up to their convictions, whilst we feel that the same cannot be said in our country. We should have

seen that in Germany the facilities given in the universities, which are Government institutions, and in the other numerous and well-organised scientific educational establishments, to original research are very great; that an original investigation in some branch of human knowledge is considered the usual termination of the student's university career; and that degrees are generally given only when some new observations or experiments have been added to the mass of human knowledge. We should find that the position of professor is mainly influenced by the amount and quality of his original researches, and that this power, and not any secondary or subsidiary ones, as is sometimes the case with us, is taken as the proof of a man's fitness to fill the professorial chair.

It is my wish, however engrossing this view of the subject may be, to ask you to consider to-day another aspect of the question—viz. the educational value of original research; the value of personal communication with nature for its own sake, the influence which such employment exerts on the mind, the effect which such studies produce as fitting men for the active duties of life, and the question, therefore, as to how far original investigation should be encouraged as an instrument of intellectual progress. It may be well, however, before we commence this special question, to place clearly before our minds what is meant by scientific inquiry in general, and to see how it is related to the studies and habits of mind with which men up to the dawn of the present, or scientific age, have been familiar.

In the first place, then, the essence of the scientific spirit is that it is free and disinterested. If, therefore, any of the habits of mind, studies, or beliefs in which men have hitherto indulged have not been free nor disinterested, in so far they have not been scientific. In the second place, the spirit of true scientific inquiry knows nothing of tradition or authority. It lays down laws for itself, and refuses to be bound by any others. Scientific education begins with no preconceived idea in accordance with which everything else must be moulded. It starts in simple communion with Nature, and is content to pick up little by little the truth which she is always ready to communicate to patient listeners. Thus step by step and generation by generation, slowly but surely, the perfect edifice of science is being built up, and all those who contribute, however insignificantly, to this great work have the safe assurance that their labour has not been in vain. This process, it is clear, at once opposed to, and, if successfully carried out, subversive of the old order of things. Between a system based on authority and one founded on freedom of thought and opinion there can never be any united action; and whilst fully acknowledging that intellectual eminence, and, of course, moral excellence, are common to all classes of men, and are not confined to those holding particular opinions, if only they be honest, it is as well that we should admit with equal candour that the followers of the old system have no claim to be called scientific, and that there is, from the nature of things, a great and impassable gulf between us and them.

It does not concern us at present to inquire which of these two systems, the free or the authoritative, is for the future to rule the world. It must now suffice for us to see clearly that the habits of mind necessary for the establishment of the one are absolutely opposed to those needed for the success of the other.

I must, however, here not be misunderstood. It would ill become me, connected as I am with a college to which it has been our constant aim to impart a university character, to undervalue or depreciate the study of subjects other than those included under the head of the physical sciences. Literary studies, whether of modern or ancient authors, giving an acquaintance with the noblest thoughts and opinions of the great men of past ages; historical studies, giving us a knowledge of the acts of men in times gone by; the study of language and philology, as giving a knowledge of how men of all times and countries express their ideas and language; of logic, as pointing out the laws of thought; and above all, that of mathematics, are all matters of the highest importance, the neglect of which would render our education poor and incomplete indeed. The same rules, however, which all acknowledge to be necessary for the teaching of physical science must be applied to the study of all these subjects. In short, the *scientific method* must be employed in all cases and carried out to its fullest extent. Whilst attempts to shackle the mind, or to stifle free inquiry, which have too frequently succeeded in past times, and which may, if we are not on our guard, succeed again, must be repulsed with all our vigour.

* Address by Prof. Roscoe at the opening of the new buildings of the Owens College Manchester.